

Why Is There Something Rather Than Nothing? A Logical Investigation *

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Abstract

From Leibniz to Krauss philosophers and scientists have raised the question as to why there is something rather than nothing (henceforth, the Question). Why-questions request a type of explanation and this is often thought to include a deductive component. With classical logic in the background only trivial answers are forthcoming. With free logics in the background, be they of the negative, positive or neutral variety, only question-begging answers are to be expected. The same conclusion is reached for the modal version of the Question, namely ‘Why is there something contingent rather than nothing contingent?’ (except that possibility of answers with neutral free logic in the background is not explored). The categorial version of the Question, namely ‘Why is there something concrete rather than nothing concrete?’, is also discussed. The conclusion is reached that deductive explanations are question-begging, whether one works with classical logic or positive or negative free logic. I also look skeptically at the prospects of giving causal-counterfactual or probabilistic answers to the Question, although the discussion of the options is less comprehensive and the conclusions are more tentative. The meta-question, viz. ‘Should we not stop asking the Question’, is accordingly tentatively answered affirmatively.

Keywords Existence; Nothingness; Why-questions; Explanations; Free Logic

1 Introduction: The Question and Logic

The central question of this article (henceforth called the Question) is: why there is *something* rather than nothing? The following modal question (henceforth called the Modal Question) will also be considered: why is there something *contingent* rather than nothing contingent? These were the questions originally asked by Leibniz (1714). Another question that is of a more recent origin is the following question (henceforth called the Categorial Question): why is there something *concrete* rather than nothing concrete?

Ever since Leibniz raised the Question and the Modal Question, philosophers continue to reflect on these questions and variations on them and this remains true of contemporary times – see e.g. (Sommers, 1966; Fleming, 1988; Van Inwagen and Lowe, 1996; Carlson and Olsson, 2001; Rundle, 2004; Parfit, 2004; Grünbaum, 2004; Maitzen, 2012; Goldschmidt, 2013). That being said, several philosophical participants to the debate, notably Grünbaum (2004) and Maitzen (2012), have come to the conclusion that it is an ill-posed question. Grünbaum (2004) calls into

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question the so-called ‘spontaneity of nothingness’, which shifts the burden of explanation to there being something. He is in favour of asking the counterquestion as to why there should be nothing (contingent) rather than something (contingent). Maitzen (2012) calls into question the determinateness of the question, because he thinks that ‘thing’ is a so-called dummy sortal for which instances there are no clear criteria of identity. This article will also present a skeptical outlook, albeit from a different perspective than Grünbaum (2004)’s or Maitzen (2012)’s.

A recent development is that some scientists have also started to think about the Question. What is more, they claim to have answered the Question (Krauss, 2012; Mlodinow and Hawking, 2010). Here is Richard Dawkins, who wrote the afterword of (Krauss, 2012):

Even the last remaining trump card of the theologian, ‘Why is there something rather than nothing?’, shrivels up before your eyes as you read these pages. If ‘On the Origin of Species’ was biology’s deadliest blow to supernaturalism, we may come to see ‘A Universe From Nothing’ [(Krauss, 2012)] as the equivalent from cosmology. The title means exactly what it says. And what it says is devastating.

The alleged answer to the Question invokes the scientific hypothesis that seemingly empty space is pervaded by quantum fields that carry energy. Albert (2012) rightly criticizes this attempt at answering the Question:

Relativistic-quantum-field-theoretical vacuum states – no less than giraffes or refrigerators or solar systems – are particular arrangements of elementary physical stuff. The true relativistic-quantum-field-theoretical equivalent to there not being any physical stuff at all isn’t this or that particular arrangement of the fields – what it is (obviously, and ineluctably, and on the contrary) is the simple absence of the fields! The fact that some arrangements of fields happen to correspond to the existence of particles and some don’t is not a whit more mysterious than the fact that some of the possible arrangements of my fingers happen to correspond to the existence of a fist and some don’t. And the fact that particles can pop in and out of existence, over time, as those fields rearrange themselves, is not a whit more mysterious than the fact that fists can pop in and out of existence, over time, as my fingers rearrange themselves. And none of these poppings – if you look at them aright – amount to anything even remotely in the neighborhood of a creation from nothing.

Albert’s point, namely that in attempting to answer the Question Krauss already presupposes that something exists, will be generalized in the present article. I don’t intend to criticize the *particular* naturalistic answer that has been put forward. I intend to criticize *any* answer that satisfies certain conditions. And my criticism is in spirit very similar to the point made by Albert.

In this article the *main* but not exclusive focus is on a logical investigation into the Question. For this purpose one needs to have a rough idea about the logic of why-questions, since the Question is a why-question. Building on Hempel and Oppenheim (1948)’s classical theory about explanations, Bromberger (1966, p. 604) developed what appears to be the first modern theory about why-questions. Let us apply his theory of why-questions to the why-question at hand. The question has a presupposition, viz. that there is something rather than nothing. A sentence is only then an answer to the question why there is something if the presupposition is deducible from the sentence together with other true premises. Bromberger (1966)’s specific account was criticized by Teller (1974) along similar lines as the well-known criticism of Hempel and Oppenheim (1948)’s theory of explanation. Later, Hintikka and Halonen (1995) and Schurz (2005) developed their own theories about why-questions in turn. The details of their theories differ, but each defends the idea that answers to why-questions stand in a deductive relation to what an explanation is asked for – see e.g. (Hintikka and Halonen, 1995, p. 648) and (Schurz,

2005, 171-172). So, whatever else needs to be satisfied in order to answer the Question, at the very least one needs a set of true premises from which one can logically deduce that something exists. The former is the most prominent approach to why-questions in the literature, but it is not the only one. Notably Koura (1988) investigates non-deductive explanations as answers to why-questions. In particular, he studies causal and probabilistic explanations that serve as answers to why-questions.

As noted by Salmon (1992) it is disputed whether every kind of explanation can function as an answer to a why-question. Still, it is worth to have a brief look at the theories of explanation, even if perhaps not every kind of explanation will do as an answer to a why-question. Some might think that the extant literature on why-questions is too much influenced by an outdated model of scientific explanation, namely Hempel and Oppenheim (1948)'s deductive-nomological theory of scientific explanation. But the latter is not the only theory of explanation that has a deductive element in it. A prominent contemporary theory about explanation is the unification approach put forward by Kitcher (1981, 1989). Roughly, an event is explained by deducing it from the most unifying scientific theory. Another example is the kairetic account introduced by Strevens (2004, 2008), according to which a causal model explains an event only if the event is entailed by it. To be sure, there are accounts of scientific explanation that are not deductive. But the important point is that there still are prominent approaches to explanation that do contain deductive elements. On this point there is no big disconnect between the literature on why-questions on the one hand and the literature on (scientific) explanations on the other hand. Nevertheless, it is also important to discuss the prospects of non-deductive answers to the Question.

Let us assume for now that answers to why-questions involve deductive arguments and, therefore, depend on logic. This brings me to investigate the relation between the Question on the one hand and logic on the other hand. In Section 2 I look at the relation between the Question and the canonical theory about logical deduction, to wit classical first-order logic with identity. In Section 3 I scrutinize the relation between the Question on the one hand and free first-order logic with identity on the other hand. Section 4 is devoted to the Modal Question and the Categorical Question. These questions also studied from both the perspective of classical logic and free logic. Although the main focus of this article is on deductive answers to the Question, I will briefly comment on causal and probabilistic answers in Section 5. Finally, in Section 6 I summarize my findings and discuss my conclusions.

2 The Question and Classical Logic

Let us consider first-order logic with identity, $CL_{=}$. I assume familiarity with the syntax and semantics of the language of $C_{=}$, namely $\mathcal{L}_{=}$, and with proof systems for classical first-order logic with identity – see (Halbach, 2010). One of the uses of identity is to express numerosity (Halbach, 2010, ch. 8) and this is key to understand the way existence is expressed in $\mathcal{L}_{=}$. Suppose that P stands for being a Wagner opera. One can use:

$$\exists x Px; \tag{1}$$

$$\exists x \exists y (Px \wedge Py \wedge x \neq y); \tag{2}$$

$$\exists x \exists y \exists z (Px \wedge Py \wedge Pz \wedge x \neq y \wedge x \neq z \wedge y \neq z) \tag{3}$$

to express respectively that:

there is at least one Wagner opera; (4)

there are at least two Wagner operas; (5)

there are at least three Wagner operas. (6)

If one wants to abstract away from Wagner operas, one needs a universal property. Self-identity, viz. $x = x$, will do, since the law of self-identity is a theorem of the logic of identity. With the help of the predicate that expresses self-identity one can use the following sentences:

$\exists x(x = x)$; (7)

$\exists x \exists y(x = x \wedge y = y \wedge x \neq y)$; (8)

$\exists x \exists y \exists z(x = x \wedge y = y \wedge z = z \wedge x \neq y \wedge x \neq z \wedge y \neq z)$ (9)

to express respectively that:

there is at least one thing; (10)

there are at least two things; (11)

there are at least three things. (12)

Note first that the two last formulas are respectively equivalent to:

$\exists x \exists y(x \neq y)$; (13)

$\exists x \exists y \exists z(x \neq y \wedge x \neq z \wedge y \neq z)$; (14)

Second, note that $\exists x(x = x)$ is truth-conditionally equivalent to $\exists x \exists y(x = y)$. This is by way of motivating the following definition, which introduces the existence predicate, $E!$.

Definition 1 (Existence). $E!t =_{df} \exists x(x = t)$, for any term t .

Importantly, the proof principles of $CL=$ and in particular the principle of existential generalisation and the law of self-identity allow one to prove that $\exists x E!x$. In other words, that there is at least one thing is a theorem of $CL=$. But then it deductively follows from any set of premises. Consequently, none of these premises is deductively essential: the conclusion follows even without them. So, any potential answer violates a non-triviality constraint, viz. that it is only then an answer when without it one cannot deduce that there is at least one thing – see e.g. (Hintikka and Halonen, 1995, p.648). From the point of view of standard logic, the question why there is something rather than nothing can only be answered in a trivial way.

Non-trivial answers can only be forthcoming if it is not a theorem of logic that there exists at least one thing. For this purpose one should drop classical logic in favour of free logic, which is so-called because it is free of existential commitment. The relation between the Question and free logic is the topic of the next section.

3 The Question and Free Logic

There are three main varieties of free logic. Suppose that nothing exists. How does this affect the truth-value of atomic subject-predicate sentences? There are three options:

1. they are *false*;

2. they can be *true*;
3. they are *neither true nor false*.

The first variation is known as *negative* free logic, the second as *positive* free logic, the third as *neutral* free logic. The three subsections deal with each variation in turn. The emphasis will be on proof theory, not model theory, since deduction is the central notion in the logic of why-questions. Details about free logics can be found in (Nolt, 2014) and (Lehmann, 2002).

3.1 The Question and Negative Free Logic

Apart from the axiom schemes of sentential logic, the following are the axiom schemes of negative free first-order logic with identity (NFL₌):

- A1** $\phi \rightarrow \forall x\phi$, with x not free in ϕ ;
- A2** $\forall x(\phi \rightarrow \psi) \rightarrow (\forall x\phi \rightarrow \forall x\psi)$
- A3** $\forall x\phi$, if $\phi(t/x)$ is an axiom;
- A4** $\forall x\phi \rightarrow (E!t \rightarrow \phi(t/x))$;
- A5** $\forall x(x = x)$;
- A6** $t = t' \rightarrow (\phi \rightarrow \phi')$, with ϕ' identical to ϕ except that zero or more occurrences of t have been replaced by t' ;
- A7** $P(t_1, \dots, t_n) \rightarrow E!t_i$, with $1 \leq i \leq n$ and with P any n -place predicate, including the identity predicate;
- A8** $E!f(t_1, \dots, t_n) \rightarrow E!t_i$, with $1 \leq i \leq n$.

The only rule of inference is modus ponens. For convenience we will assume that no free variables occur in the above terms and formulas, except possibly x in ϕ or ψ in **A2** or in ϕ in **A3** or **A4**. Axiom scheme **A4** is characteristic for *free* logic, while axiom schemes **A7** and **A8** are characteristic for *negative* free logic. It is important to notice that one cannot prove the reverse of axiom scheme **A1**, although it is a theorem of classical logic. Especially important is that axiom scheme **A1** is equivalent to

$$\mathbf{A1}^* \quad \exists x\phi \rightarrow \phi \text{ (with } x \text{ not free in } \phi).$$

Naturally, the converse is also not provable. Also important is that axiom scheme **A4** is equivalent to:

$$\mathbf{A4}^* \quad (E!t \wedge \phi(t/x)) \rightarrow \exists x\phi.$$

Although the official proof system is the axiomatic one, the following natural deduction rules will also be used sometimes:

- VI** Given a derivation of $\phi(c/x)$ from $E!c$, where c is a new individual constant and does not occur in ϕ , discharge $E!c$ and infer $\forall x\phi$.
- VE** Given $\exists x\phi$ and a derivation of a formula ψ from $\phi(c/x) \wedge E!c$, where c is a new individual constant and does not occur in either ϕ or ψ , discharge $\phi(c/x) \wedge E!c$ and infer ψ from $\exists x\phi$.

The above rules can be derived in the axiomatic proof system.

The main result of this subsection is that any deduction with an existential conclusion (i.e., a sentence of the form $\exists x\psi$ for some ψ) starts from at least one premise that is itself existential or that is logically equivalent to an existential assumption.

Lemma 1. *For every sentence ϕ there is formula ψ with at most one free variable x such that $\vdash_{NFL=} \phi \leftrightarrow \exists x\psi$ or $\vdash_{NFL=} \phi \leftrightarrow \forall x\psi$.*

Proof. The proof is by induction on the complexity of ϕ .

Case 1: suppose that ϕ is $P(t_1, \dots, t_n)$, with P an n -place predicate (possibly the identity predicate) and with t_1, \dots, t_n terms. Then one can prove that ϕ is provably equivalent in $NFL=$ to:

$$\exists x_1 \dots \exists x_n (x_1 = t_1 \wedge \dots \wedge x_n = t_n \wedge P(x_1, \dots, x_n)).$$

For the left-to-right direction use axiom scheme **A7** to derive that

$$P(t_1, \dots, t_n) \wedge E!t_1 \wedge \dots \wedge E!t_n.$$

Use the combination of axiom schemes **A5** and **A4** to further derive that $t_1 = t_1 \wedge \dots \wedge t_n = t_n$. The conclusion follows by **A4***. For the right-to-left direction use $\exists E$ and axiom scheme **A6**.

Case 2: suppose that ϕ is $\neg\theta$. By the induction hypothesis, there is formula ψ with at most one free variable x such that $\vdash_{NFL=} \theta \leftrightarrow \exists x\psi$ or $\vdash_{NFL=} \theta \leftrightarrow \forall x\psi$. But then one can prove that $\neg\theta$ is equivalent to $\neg\exists x\psi$ or, equivalently, $\forall x\neg\psi$, or one can prove that $\neg\theta$ is equivalent to $\neg\forall x\psi$ or, equivalently, $\exists x\neg\psi$.

Case 3: suppose that ϕ is $\theta \rightarrow \rho$. We have to consider four subcases.

Case 3.1: θ is provably equivalent to $\exists x\alpha$ and ρ to $\exists x\beta$. Then ϕ is provably equivalent to $\forall x(\alpha \rightarrow \exists x\beta)$. Let us prove both directions by *reductio ad absurdum*. For the left-to-right direction suppose that $\exists x\alpha \rightarrow \exists x\beta$ but $\neg\forall x(\alpha \rightarrow \exists x\beta)$. Then $\exists x(\alpha \wedge \neg\exists x\beta)$. Proceed by $\exists E$. Suppose that $E!c \wedge \alpha(c/x) \wedge \neg\exists x\beta$. The last conjunct together with the first main assumptions entails that $\neg\exists x\alpha$. Using **A4** one can derive that $E!c \rightarrow \neg\alpha(c/x)$, which quickly leads to a contradiction. For the right-to-left direction suppose that $\forall x(\alpha \rightarrow \exists x\beta)$ but $\neg(\exists x\alpha \rightarrow \exists x\beta)$. It follows that $\exists x\alpha \wedge \neg\exists x\beta$. Continuing with $\exists E$, assume that $E!c \wedge \alpha(c/x)$. With **A4** and the first of the main assumptions one can deduce that $E!c \rightarrow (\alpha(c/x) \rightarrow \exists x\beta)$ and, hence, $\alpha(c/x) \rightarrow \exists x\beta$ and, finally, $\exists x\beta$. Contradiction.

Case 3.2: θ is provably equivalent to $\forall x\alpha$ and ρ to $\forall x\beta$. Then ϕ is provably equivalent to $\forall x(\forall x\alpha \rightarrow \beta)$. Let us prove both directions by *reductio ad absurdum*. For the left-to-right direction suppose that $\forall x\alpha \rightarrow \forall x\beta$ but $\neg\forall x(\forall x\alpha \rightarrow \beta)$. Exchanging the quantifier in the second assumption and proceeding by $\exists E$ assume that $E!c \wedge \forall x\alpha \wedge \neg\beta(c/x)$. Together with the first assumption this entails that $\forall x\beta$. By **A4** one can deduce that $E!c \rightarrow \beta(c/x)$, which quickly leads to a contradiction. For the right-to-left direction suppose that $\forall x(\forall x\alpha \rightarrow \beta)$ but $\neg(\forall x\alpha \rightarrow \forall x\beta)$. Then it follows that $\forall x\alpha \wedge \neg\forall x\beta$. Exchanging the quantifier and proceeding by $\exists E$ assume that $E!c \wedge \neg\beta(c/x)$. Next, use **A4** to derive $E!c \rightarrow (\forall x\alpha \rightarrow \beta(c/x))$ from the first main assumption. This yields $\beta(c/x)$. Contradiction.

Case 3.3: θ is provably equivalent to $\exists x\alpha$ and ρ to $\forall x\beta$. Then ϕ is provably equivalent to $\forall x(\alpha \rightarrow \forall x\beta)$. Let us prove both directions by *reductio ad absurdum*. For the left-to-right direction suppose that $\exists x\alpha \rightarrow \forall x\beta$ but $\neg\forall x(\alpha \rightarrow \forall x\beta)$. Exchanging the quantifier and proceeding by $\exists E$ assume that $E!c \wedge \alpha(c/x) \wedge \neg\forall x\beta$. The last conjunct together with the first main assumption entails that $\neg\exists x\alpha$. Exchanging the quantifier and using **A4** one can deduce that $E!c \rightarrow \neg\alpha(c/x)$, which quickly leads to a contradiction. For the right-to-left direction suppose that $\forall x(\alpha \rightarrow \forall x\beta)$ but $\neg(\exists x\alpha \rightarrow \forall x\beta)$. The latter implies that $\exists x\alpha \wedge \neg\forall x\beta$. Proceeding by

$\exists E$ assume that $E!c \wedge \alpha(c/x)$. By **A4** one can deduce from the first main assumption that $E!c \rightarrow (\alpha(c/x) \rightarrow \forall x\beta)$. So, $\forall x\beta$ and, hence, contradiction.

Case 3.4: θ is provably equivalent to $\forall x\alpha$ and ρ to $\exists x\beta$. Then ϕ is provably equivalent to $\exists x(\forall x\alpha \rightarrow \beta)$. Let us prove both directions by *reductio ad absurdum*. For the left-to-right direction suppose that $\forall x\alpha \rightarrow \exists x\beta$ but $\neg\exists x(\forall x\alpha \rightarrow \beta)$. Let us reason by cases from the first assumption. After exchanging the quantifiers the first case is $\exists x\neg\alpha$. Proceeding by $\exists E$ assume that $E!c \wedge \neg\alpha(c/x)$. Exchange the quantifiers of the second main assumption and use **A4** to derive that $E!c \rightarrow (\forall x\alpha \wedge \neg\beta(c/x))$. Therefore, $\forall x\alpha$. Use **A4** once more to derive $E!c \rightarrow \alpha(c/x)$. This quickly leads to contradiction. The second case is $\exists x\beta$. Proceeding by $\exists E$ assume that $E!c \wedge \beta(c/x)$. Use **A4** to derive that $E!c \rightarrow (\forall x\alpha \wedge \neg\beta(c/x))$. Hence, $\neg\beta(c/x)$. Contradiction. For the right-to-left direction suppose that $\exists x(\forall x\alpha \rightarrow \beta)$ but $\neg(\forall x\alpha \rightarrow \exists x\beta)$. We are going to use $\exists E$ and suppose that $E!c \wedge (\forall x\alpha \rightarrow \beta(c/x))$. It follows from the second main assumption that $\forall x\alpha \wedge \neg\exists x\beta$. Hence, $\beta(c/x)$. By **A4** it also follows, after exchanging the quantifier, that $E!c \rightarrow \neg\beta(c/x)$. Contradiction follows quickly.

Case 4: ϕ is $\forall x\theta$. This is trivial. □ □

Theorem 1. *An existential sentence can only be deduced in $NFL_{=}$ from a set of sentences Γ if at least one of the sentences in Γ is provably equivalent to an existential sentence.*

Proof. Consider a set of sentences Γ . Either all sentences in Γ are not provably equivalent to existential sentences or at least one sentence in Γ is provably equivalent to an existential sentence. In the first case it follows by Lemma 1 that they are all equivalent to universally quantified sentences. But a model with an empty domain of quantification makes all the universally quantified sentences true while making any existentially quantified sentence false. So in the first case one cannot validly deduce an existential sentence from Γ . So, if one can validly deduce an existential sentence from Γ , then at least one of the sentences of Γ is logically equivalent to an existential sentence. □ □

Whereas $CL_{=}$ has a problematic relation with the Question because it has as a theorem that there exists something, $NFL_{=}$ does not have the existential claim as a theorem but it does only yield an existential output if there is an existential input. There is no free lunch in negative free logic. I take this result to mean that answers to the Question are question-begging, because the arguments that are the explanations are question-begging. According to Jacquette (1993, p. 319, 322), an argument A is question-begging if and only if

- (1) [...] A contains premise P and conclusion C, and P presupposed C.
- (2') P presupposes C if and only if it is not justified to believe P unless it is justified to believe C.

Similarly, Fischer and Pendergraft (2013, p. 584) claim that

[...] an argument begs the question just in case the proponent of the argument has no reason to accept the relevant premise, apart from a prior acceptance of the conclusion.

I claim that these conditions are fulfilled in the case of deductive arguments for the existence of something. If such arguments have to start from at least one premise that is itself existential or logically equivalent to an existential assumption, then that premise is only justified if the existential conclusion is justified.

3.2 The Question and Positive Free Logic

The axiomatic theory laid out in the previous subsection contains the core of positive free logic as well: axiom schemes **A1-A4**, **A6** are retained, but **A5** is replaced by

$$\mathbf{A5}^* \quad t = t$$

and **A7-A8** are dropped. Axiom scheme **A5*** is characteristic for *positive* free logic.

Since **A7** has been dropped, one cannot get the proof of a result similar to Lemma 1 off the ground: the base case depends essentially on **A7**.

The main result of this subsection is that any deduction with an existential conclusion starts from at least one premise that is itself existential or it starts from premises the conjunction of which is logically equivalent to an existential assumption.

Theorem 2. *An existential sentence can only be deduced in $PFL_{=}$ from a set of sentences Γ if at least one of the sentences in Γ is an existential sentence or there are sentences in Γ such that their conjunction is logically equivalent to an existential sentence.*

Proof. The proof is by induction on $\Gamma \vdash_{PFL_{=}} \exists x\phi$, with ϕ a formula with at most x free.

Case 1: no existential sentence is a logical axiom of $PFL_{=}$.

Case 2: if $\exists x\phi \in \Gamma$, then the condition holds, since $\exists x\phi$ is an existential sentence.

Case 3: there is a ψ such that $\Gamma \vdash_{PFL_{=}} \psi$ and $\Gamma \vdash_{PFL_{=}} \psi \rightarrow \exists x\phi$. The claim is that $\psi \wedge (\psi \rightarrow \exists x\phi)$ is logically equivalent to an existential sentence. In what follows keep in mind that we are dealing with sentences here. First, note that, given $\psi \rightarrow \exists x\phi$, it also follows that $\psi \rightarrow \exists x\psi$. Since $\psi \rightarrow \exists x\phi$ is logically equivalent to $\neg\psi \vee \exists x\phi$, we can argue by cases. Indeed, if $\neg\psi$, then $\psi \rightarrow \exists x\psi$, and if ψ , then $\exists x\phi$ and so by $\exists\mathbf{E}$ and **A4*** $\psi \rightarrow \exists x\psi$ as well. Second, given the previous result and **A1***, it follows that $\Gamma \vdash_{PFL_{=}} \psi \leftrightarrow \exists x\psi$. Third, $\exists x\psi \wedge (\psi \rightarrow \exists x\phi)$ logically entails $\exists x(\psi \wedge (\psi \rightarrow \exists x\phi))$. The proof is by $\exists\mathbf{E}$ and axiom scheme **A4***. Therefore, $\exists x(\psi \wedge (\psi \rightarrow \exists x\phi))$ is entailed by $\psi \wedge (\psi \rightarrow \exists x\phi)$. Given **A1***, $\exists x(\psi \wedge (\psi \rightarrow \exists x\phi))$ entails $\psi \wedge (\psi \rightarrow \exists x\phi)$.

Of course, ψ or $\psi \rightarrow \exists x\phi$ may not belong to Γ . Then there is a finite (and possibly empty) set of sentences $\alpha_1, \dots, \alpha_n$ such that $\alpha_1, \dots, \alpha_n \in \Gamma$ and

$$\alpha_1, \dots, \alpha_n \vdash_{PFL_{=}} \psi.$$

Furthermore, there is a finite (and non-empty) set of sentences β_1, \dots, β_m such that $\beta_1, \dots, \beta_m \in \Gamma$ and

$$\beta_1, \dots, \beta_m \vdash_{PFL_{=}} \psi \rightarrow \exists x\phi.$$

Therefore,

$$\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m \vdash_{PFL_{=}} \psi \wedge (\psi \rightarrow \exists x\phi).$$

As we have seen, the conclusion is logically equivalent to $\exists x(\psi \wedge (\psi \rightarrow \exists x\phi))$. It is a consequence that

$$\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m \vdash_{PFL_{=}} \exists x(\psi \wedge (\psi \rightarrow \exists x\phi)).$$

Hence,

$$\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m \vdash_{PFL_{=}} \alpha_1 \rightarrow \exists x(\psi \wedge (\psi \rightarrow \exists x\phi)).$$

By familiar reasoning, it follows that

$$\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m \vdash_{PFL_{=}} \alpha_1 \rightarrow \exists x\alpha_1,$$

which entails that

$$\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m \vdash_{PFL_{=}} \exists x\alpha_1,$$

and therefore also

$$\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m \vdash_{PFL=} \exists x \alpha_1 \wedge \dots \wedge \alpha_n \wedge \beta_1 \wedge \dots \wedge \beta_m.$$

The existence quantifier distributes over *sentences*, so the conclusion is that

$$\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m \vdash_{PFL=} \exists x (\alpha_1 \wedge \dots \wedge \alpha_n \wedge \beta_1 \wedge \dots \wedge \beta_m).$$

The other direction can be proved by axiom scheme **A1***. □

Corollary 1. *An existential sentence can only be deduced in $PFL=$ from a set of sentences Γ if and only if it can be deduced from a set of sentences Γ^* that contains at least one existential sentence or a sentence that is logically equivalent to an existential sentence.*

The philosophical import of Corollary 1 is again taken to be that any purported answer to the Question is question-begging.

3.3 The Question and Neutral Free Logic

The third and final variation is neutral free logic ($NEFL=$). In fact, there are quite a few options one can take, depending on how one wants to calculate the truth-value of formulas that have subformulas that are neither true nor false. For the sake of convenience, I will restrict myself to the Fregean option: complex formulas that have subformulas that are neither true nor false are themselves neither true nor false. The proof theory is quite different from $NFL=$ and $PFL=$. I will briefly describe the tree proof system developed by Lehmann (2002, 235-237).

A marker $*$ is added to $\mathcal{L}_=$. If ϕ is a well-formed formula of $\mathcal{L}_=$, then ϕ^* is a well-formed formula of $\mathcal{L}_=^*$. Think of the star marker as an indicator that the formula has a determinate truth value. An elementary formula is an atomic formula or its negation. The tree proof rules can be found in Table 1.¹

If α is a quantified formula or the negation thereof, or an elementary formula, each term of which occurs in an elementary $*$ -formula above α^* , then the following tree proof rule applies:

$$\frac{\alpha}{\alpha^*}$$

A branch closes if and only if

1. it contains a formula and its negation and at least one of those two formulas is a $*$ -formula, or
2. it contains $t \neq t^*$ for some term t .

A tree is closed if and only if each of its branches is closed.

One can distinguish between three different deducibility relations:

1. $\Gamma \vdash_1 \phi$ iff the tree starting with Γ^* (i.e., $\{\phi^* \mid \phi \in \Gamma\}$) and $\neg\phi^*$ closes;
2. $\Gamma \vdash_{2a} \phi$ iff the tree starting with Γ^* and $\neg\phi$ closes;
3. $\Gamma \vdash_{2b} \phi$ iff the tree starting with Γ and $\neg\phi^*$ closes.

¹Lehmann (2002) did not provide rules for *vacuously* quantified sentences.

| | |
|---|---|
| $\begin{array}{c} \neg\neg\alpha \\ \\ \alpha \end{array}$ | $\begin{array}{c} \neg\neg\alpha^* \\ \\ \alpha^* \end{array}$ |
| $\begin{array}{c} \alpha \rightarrow \beta \\ \swarrow \searrow \\ \neg\alpha \quad \beta \end{array}$ | $\begin{array}{c} (\alpha \rightarrow \beta)^* \\ \swarrow \quad \quad \searrow \\ \alpha^* \quad \neg\alpha^* \quad \neg\alpha^* \\ \beta^* \quad \beta^* \quad \neg\beta^* \end{array}$ |
| $\begin{array}{c} \neg(\alpha \rightarrow \beta) \\ \swarrow \quad \quad \searrow \\ \alpha \quad \alpha \quad \beta \\ \neg\beta \quad \neg\alpha \quad \neg\beta \end{array}$ | $\begin{array}{c} \neg(\alpha \rightarrow \beta)^* \\ \\ \alpha^* \\ \neg\beta^* \end{array}$ |
| $\begin{array}{c} \alpha(s) \\ s = t^* \\ \\ \alpha(t) \end{array}$ | $\begin{array}{c} \alpha(s)^* \\ s = t^* \\ \\ \alpha(t)^* \end{array}$ |
| if α is an elementary formula | |
| $\begin{array}{c} \forall x\alpha(x)^* \\ \\ \alpha(t) \end{array}$ | $\begin{array}{c} \neg\exists x\alpha(x)^* \\ \\ \neg\alpha(t) \end{array}$ |
| if t occurs in an elementary $*$ -formula above $\alpha(t)$ or $\neg\alpha(t)$ | |
| $\begin{array}{c} \neg\forall x\alpha(x)^* \\ \\ y = y^* \\ \neg\alpha(y)^* \end{array}$ | $\begin{array}{c} \exists x\alpha(x)^* \\ \\ y = y^* \\ \alpha(y)^* \end{array}$ |
| if y does not occur free above $y = y^*$ | |

Table 1: Tree proof rules for NEFL₌

The first deducibility relation is supposed to correspond with inferences that do not lead from true premises to false conclusions. The second deducibility relation is supposed to correspond with truth-preserving inferences, while the third deducibility relation is supposed to correspond to inferences that preserve non-falsehood. The notion of deducibility that is most relevant here is the second one. For theories about deductive explanations hold that an *explanans* has to be true. Therefore, an *explanans* has to have a determinate truth value, which is syntactically indicated by the star. This rules out the third notion of deducibility. In favour of using the second notion and not the first notion is that the notion of provable equivalence that can be defined with it has two useful properties that it otherwise would not have. First, it allows the substitution of a true sentence in the premise set with another sentence that is provably equivalent to it and, hence, is true as well. Second, it allows the substitution of a subsentence of a starred sentence, which itself also has a determinate truth value, with another sentence that is provably equivalent to it and, hence, has the same determinate truth value. Both properties will be used below.

With the proof theory in place we are ready to prove the following crucial lemma:

Lemma 2. *For every sentence ϕ there is a formula ψ with one free variable x such that $\phi \vdash_{2a} \vdash_{2a} \exists x \psi(x)$ or $\phi \vdash_{2a} \vdash_{2a} \forall x \psi(x)$.*

Proof. The proof is by induction on the complexity of ϕ . The tree proofs for all the claimed equivalences can be found in Appendix A.

Case 1-i: ϕ is $P(t_1, \dots, t_n)$, with P an n -place predicate (possibly the identity predicate) and with t_1, \dots, t_n terms. The sentence in question logically entails

$$\exists x_1 \dots \exists x_n (x_1 = t_1 \wedge \dots \wedge x_n = t_n \wedge P(x_1 \wedge \dots \wedge x_n)).$$

Case 1-ii: ϕ is $\neg P(t_1, \dots, t_n)$, with P an n -place predicate (possibly the identity predicate) and with t_1, \dots, t_n terms. The sentence in question logically entails

$$\exists x_1 \dots \exists x_n (x_1 = t_1 \wedge \dots \wedge x_n = t_n \wedge \neg P(x_1 \wedge \dots \wedge x_n)).$$

Case 2: ϕ is $\neg\neg\psi$. Given the induction hypothesis and the rules for double negation, this is trivial.

Case 3-i: ϕ is $(\alpha \rightarrow \beta)$. There are four subcases to consider.

Case 3.1-i: ϕ is $(\exists x \psi_1(x) \rightarrow \exists x \psi_2(x))$. The latter is provably equivalent to

$$\forall x (\psi_1(x) \rightarrow \exists x \psi_2(x)).$$

Case 3.2-i: ϕ is $(\forall x \psi_1(x) \rightarrow \forall x \psi_2(x))$. The latter is provably equivalent to

$$\forall x (\forall x \psi_1(x) \rightarrow \psi_2(x)).$$

Case 3.3-i: ϕ is $(\exists x \psi_1(x) \rightarrow \forall x \psi_2(x))$. The latter is provably equivalent to

$$\forall x (\psi_1(x) \rightarrow \forall x \psi_2(x)).$$

Case 3.4-i: ϕ is $(\forall x \psi_1(x) \rightarrow \exists x \psi(x))$. The latter is provably equivalent to

$$\exists x ((\forall x \psi_1(x) \rightarrow \exists x \psi(x)) \wedge x = x).$$

Case 3-ii: ϕ is $\neg(\alpha \rightarrow \beta)$. There are four subcases to consider.

Case 3.1-ii: ϕ is $\neg(\exists x \psi_1(x) \rightarrow \exists x \psi_2(x))$. The latter is provably equivalent to

$$\exists x \neg(\psi_1(x) \rightarrow \exists x \psi_2(x)).$$

Case 3.2-ii: ϕ is $\neg(\forall x\psi_1(x) \rightarrow \forall x\psi_2(x))$. The latter is provably equivalent to

$$\exists x\neg(\forall x\psi_1(x) \rightarrow \psi_2(x)).$$

Case 3.3-ii: ϕ is $\neg(\exists x\psi_1(x) \rightarrow \forall x\psi_2(x))$. The latter is provably equivalent to

$$\exists x\neg(\exists x\psi_1(x) \rightarrow \psi_2(x)).$$

Case 3.4-ii: ϕ is $\neg(\forall x\psi_1(x) \rightarrow \exists x\psi(x))$. The latter is provably equivalent to

$$\forall x((\forall x\psi_1(x) \rightarrow \exists x\psi(x)) \wedge x = x).$$

Case 4-i: ϕ is $\exists x\psi(x)$ or $\forall x\psi(x)$. This is trivial.

Case 4-ii: ϕ is $\neg\exists x\psi(x)$ or $\neg\forall x\psi(x)$. It is provable that $\neg\exists x\psi(x)$ is equivalent to $\forall x\neg\psi(x)$ and that $\neg\forall x\psi(x)$ is equivalent to $\exists x\neg\psi(x)$.

Again, the tree proofs for all the claimed equivalences can be found in Appendix A.

□

□

Theorem 3. *An existential sentence can only be deduced in $NEFL_{=}$ (in the sense of \vdash_1 or \vdash_{2a}) from a set of sentences Γ if at least one of the sentences in Γ is an existential sentence or logically equivalent (in the sense of \vdash_{2a}) to one.*

Proof. For any sentence $\phi \in \Gamma$ there is a formula ψ with one free variable such that $\phi \vdash_{2a} \vdash_{2a} \exists x\psi(x)$ or $\phi \vdash_{2a} \vdash_{2a} \forall x\psi(x)$ (Lemma 2). Suppose that they are all provably equivalent to universally quantified sentences only. Then an existential sentence cannot be deduced. The only rule that can be applied to universally quantified sentences is the instantiation rule. But that rule can only be applied if there is a term t that occurs in an elementary $*$ -sentence higher up. In the proof of Lemma 2 it was shown that elementary $*$ -sentences are provably equivalent to existentially quantified sentences. By contraposition, a valid deduction of an existential sentence from a set of starred sentences can only happen if at least one of the starred sentences is logically equivalent to an existential sentence. □

□

The philosophical lesson of Theorem 3 is once again that any potential answer to the Question is question-begging.

4 The Modal Question and the Categorical Question

One might object that the logical investigation of the Question is nice, but the Question is not the real question. Rather the real question is the Modal question, viz. ‘Why is there something contingent rather than nothing contingent?’. Or maybe the better question is the Categorical Question, viz. ‘Why is there something concrete rather than nothing concrete?’. Let us discuss these more restricted questions in turn.

To carry out a logical investigation into the Modal Question, we need to consider a first-order language with identity and a necessity operator \Box , viz. $\mathcal{L}_{=,\Box}$. It has the expressive resources to express that something contingent exists:

$$\exists x(E!x \wedge \neg\Box E!x). \quad (15)$$

Note that the above is logically equivalent to $\exists x\neg\Box E!x$, regardless of whether one uses classical logic or free logic.

With classical first-order logic with identity and the weakest normal modal system, **K**, in the background one can prove that everything has necessary existence, expressed by $\forall x\Box E!x$

or $\forall x \Box \exists y (x = y)$ – see (Menzel, 2014). This makes the presupposition of the Modal Question *logically false*. Therefore, no sound argument for the presupposition is forthcoming.

The situation is different with free modal logic. Let us call it $NFL_{=,\Box}$ or $PFL_{=,\Box}$, depending on whether it is an extension of negative free logic or positive free logic. The latter are extended with modal system **S5**. For more on axiomatic modal free logic, see (Hughes and Cresswell, 1996, p. 293-296).² Necessary existence is no longer a theorem. So the presupposition of the Modal Question is not logically false. Still, one can prove a result analogous to Theorem 2.

Theorem 4. *A sentence of the form $\exists x (\neg \Box E!x \wedge \phi)$ can only be deduced in $N/PFL_{=,\Box}$ from a set of sentences Γ if at least one of the sentences in Γ is a sentence of the form $\exists x (\neg \Box E!x \wedge \psi)$ or there are sentences in Γ such that their conjunction is provably equivalent to a sentence of the form $\exists x (\neg \Box E!x \wedge \psi)$.*

Proof. The proof is by induction on $\Gamma \vdash_{N/PFL_{=,\Box}} \exists x (\neg \Box E!x \wedge \phi)$, with ϕ a formula with at most x free.

Case 1: no sentence of the form $\exists x (\neg \Box E!x \wedge \phi)$ is a logical axiom of $N/PFL_{=,\Box}$.³

Case 2: if $\exists x (\neg \Box E!x \wedge \phi) \in \Gamma$, then the condition holds.

Case 3:⁴ there is a ψ such that $\Gamma \vdash_{N/PFL_{=,\Box}} \psi$ and

$$\Gamma \vdash_{N/PFL_{=,\Box}} \psi \rightarrow \exists x (\neg \Box E!x \wedge \phi).$$

First, note that, given $\psi \rightarrow \exists x (\neg \Box E!x \wedge \phi)$, it also follows that

$$\psi \rightarrow \exists x (\neg \Box E!x \wedge \psi).$$

Since $\psi \rightarrow \exists x (\neg \Box E!x \wedge \phi)$ is logically equivalent to $\neg \psi \vee \exists x (\neg \Box E!x \wedge \phi)$, we can argue by cases. Indeed, if $\neg \psi$, then $\psi \rightarrow \exists x (\neg \Box E!x \wedge \psi)$, and if ψ , then by the fact that ψ is a sentence, $\exists E$ and **A4*** $\psi \rightarrow \exists x (\neg \Box E!x \wedge \psi)$ follows as well. Second, given the previous result, the fact that ψ is a sentence and $\exists E$, it follows that $\Gamma \vdash_{N/PFL_{=,\Box}} \psi \leftrightarrow \exists x (\neg \Box E!x \wedge \psi)$. Third, $\exists x (\neg \Box E!x \wedge \psi) \wedge (\psi \rightarrow \exists x (\neg \Box E!x \wedge \phi))$ provably implies

$$\exists x (\neg \Box E!x \wedge \psi \wedge (\psi \rightarrow \exists x (\neg \Box E!x \wedge \phi))).$$

The proof is by $\exists E$ and axiom scheme **A4***. Therefore,

$$\exists x (\neg \Box E!x \wedge \psi \wedge (\psi \rightarrow \exists x (\neg \Box E!x \wedge \phi)))$$

is entailed by $\psi \wedge (\psi \rightarrow \exists x (\neg \Box E!x \wedge \phi))$. The other direction holds because of $\exists E$ and the fact that $\psi \wedge (\psi \rightarrow \exists x (\neg \Box E!x \wedge \phi))$ is a sentence.

²There is one important difference between $N/PFL_{=,\Box}$ on the one hand and $LPCE + \mathbf{S5}$, the system in (Hughes and Cresswell, 1996), on the other hand: $\phi \leftrightarrow \forall x \phi$ (provided that x is not free in ϕ) is an axiom scheme of $LPCE + \mathbf{S5}$, whereas only the left-to-right direction is an axiom scheme of $N/PFL_{=,\Box}$. Semantically, the difference is that in $N/PFL_{=,\Box}$ the world-relative domains of quantification can be empty, whereas they cannot in $LPCE + \mathbf{S5}$. The formal relevance of **S5** consists in the fact that one does not need to assume a certain primitive rule called $UGL\forall^n$. The material relevance of **S5** is due to the fact that it is generally taken to the correct logic for metaphysical or counterfactual necessity – see (Williamson, 2013) for an argument. The dialectical relevance of **S5** is that it gives very strong modal resources to those who attempt a deductive explanation.

³Note that, even if $\phi \leftrightarrow \forall x \phi$ (with x not free in ϕ) were one of the axiom schemes (as in $LPCE + \mathbf{S5}$ – see footnote 4), this would still hold.

⁴Hughes and Cresswell (1996, p. 293) mention three other inference rules. For one of these, see note 4. The two other rules are the rule of necessitation (if $\vdash_{N/PFL_{=,\Box}} \phi$, then $\vdash_{N/PFL_{=,\Box}} \Box \phi$) and the rule of universal generalisation (if $\vdash_{N/PFL_{=,\Box}} \phi$, then $\vdash_{N/PFL_{=,\Box}} \forall x \phi$). For the two others, note the following two things. First, they can be made redundant, e.g. one can stipulate that all the axioms are necessary. (Necessity is closed under modus ponens.) Second, in neither case is the conclusion of the inference of the right syntactic form.

Of course, ψ or $\psi \rightarrow \exists x(\neg \Box E!x \wedge \phi)$ may not belong to Γ . Then there is a finite (and possibly empty) set of sentences $\alpha_1, \dots, \alpha_n$ such that $\alpha_1, \dots, \alpha_n \in \Gamma$ and

$$\alpha_1, \dots, \alpha_n \vdash_{N/PFL=\Box} \psi.$$

Furthermore, there is a finite (and non-empty) set of sentences β_1, \dots, β_m such that $\beta_1, \dots, \beta_m \in \Gamma$ and

$$\beta_1, \dots, \beta_m \vdash_{N/PFL=\Box} \psi \rightarrow \exists x(\neg \Box E!x \wedge \phi).$$

Therefore,

$$\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m \vdash_{FL=\Box} \psi \wedge (\psi \rightarrow \exists x(\neg \Box E!x \wedge \phi)).$$

As we have seen, the conclusion is provably equivalent to

$$\exists x(\neg \Box E!x \wedge \psi \wedge (\psi \rightarrow \exists x(\neg \Box E!x \wedge \phi))).$$

It is a consequence that

$$\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m \vdash_{N/PFL=\Box} \exists x(\neg \Box E!x \wedge \psi \wedge (\psi \rightarrow \exists x(\neg \Box E!x \wedge \phi))).$$

Hence,

$$\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m \vdash_{N/PFL=\Box} \alpha_1 \rightarrow \exists x(\neg \Box E!x \wedge \psi \wedge (\psi \rightarrow \exists x(\neg \Box E!x \wedge \phi))).$$

By familiar reasoning, it follows that

$$\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m \vdash_{N/PFL=\Box} \alpha_1 \rightarrow \exists x(\neg \Box E!x \wedge \alpha_1),$$

which entails that

$$\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m \vdash_{N/PFL=\Box} \exists x(\neg \Box E!x \wedge \alpha_1).$$

Since $\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m$ are sentences and since one has $\exists \mathbf{E}$, the conclusion is that

$$\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m \vdash_{FL=\Box} \exists x(\neg \Box E!x \wedge \alpha_1 \wedge \dots \wedge \alpha_n \wedge \beta_1 \wedge \dots \wedge \beta_m).$$

The other direction follows directly from $\exists \mathbf{E}$ and the assumption that

$$\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m$$

are sentences.⁵

□

□

With free logic of either the positive or negative flavour in the background, it is not only the Question that can only receive question-begging answers, but also the Modal Question can only receive question-begging answers. For neutral free logic one would need to have rules for $\Box \phi^*$ and $\neg \Box \phi^*$, but I am not going to pursue that option here.

To carry out a logical investigation into the Categorical Question, we need to consider a first-order language with identity, the necessity operator \Box and a concreteness predicate C , viz. $\mathcal{L}_{=\Box, C}$. It has the expressive resources to express that something concrete exists:

$$\exists x(E!x \wedge C(x)). \tag{16}$$

Note that the above is logically equivalent to $\exists x C(x)$, regardless of whether one uses classical logic or free logic.

⁵Note that the proved equivalences in Case 3 are unaffected even if one were to add $\phi \leftrightarrow \forall x \phi$ (with x not free in ϕ) as an axiom scheme.

Some philosophers embrace the necessity of existence ($\forall x \Box E!x$), which is provable in classical-first order logic with identity and modal system **K** (Linsky and Zalta, 1994; Williamson, 2013). They think that the necessity of existence is defensible, as long as one is careful not to interpret the quantifiers as ranging over concrete objects only and, if the domain of quantification does contain concrete objects, then one should allow objects to be contingently concrete. Coming from this angle one may want to reformulate the Categorical Question as follows: why does anything *contingently concrete* exist rather than nothing? In other words, one requests an explanation for

$$\exists x (E!x \wedge C(x) \wedge \neg \Box C(x)), \quad (17)$$

which again can be simplified to $\exists x (C(x) \wedge \neg \Box C(x))$. It is for this reason that I consider a language that contains the necessity operator as well.

Now let me make two observations. First, one can prove a theorem that is completely analogous to Theorem 4.

Theorem 5. *A sentence of the form $\exists x (C(x) \wedge \phi)$ can only be deduced in $N/PFL_{=, \Box, C}$ from a set of sentences Γ if at least one of the sentences in Γ is a sentence of the form $\exists x (C(x) \wedge \psi)$ or there are sentences in Γ such that their conjunction is provably equivalent to a sentence of the form $\exists x (C(x) \wedge \psi)$.*

Proof. Run through the proof of Theorem 4 and systematically replace $\neg \Box E!x$ with $C(x)$. $\square \quad \square$

Second, unlike with the Question and the Modal Question one does not get trivialisation if classical logic in the background, but one can expect the deductive explanations to be question-begging. Note that one can obtain classical logic from free logic by adding $\exists x E!x$ and $E!t$ to the axioms. Neither of these has the form $\exists x (C(x) \wedge \phi)$. Since classical logic is an extension of free logic and since $\exists x (C(x) \wedge \phi)$ still does not belong to the axiomatic base, Theorem 5 applies as well.

5 The Question and Causal and Probabilistic Answers

So far we have been assuming that answers to why-questions are deductive arguments and, hence, depend on logic. Let us now drop that assumption and have a brief look at non-deductive answers. Koura (1988) studies two main alternatives, namely causal answers and probabilistic answers. Let us discuss them in turn.

The first main alternative consists in causal answers. As was already mentioned in Section 1, there are variations of the causal approach that *are* deductive as well, notably the kairetic account of Strevens (2004, 2008). But here the focus has to be on non-deductive causal theories. Also, some of the theories about causation are probabilistic in nature, notably the causal-relevance model of Salmon (1971). This also has to be put aside. What is left is the counterfactual approach to causation, which goes back to Lewis (1973a, 1986), but which nowadays comes in different shapes (Woodward and Hitchcock, 2003). Surveying all the possibilities would considerably lengthen this article, so I propose to have a look at the simplest counterfactual theory of causation (Lewis, 1973a): an event C causally depends on an event E if and only if, had C not occurred, E would not have occurred, and if C had occurred, E would have occurred. Let $\Box \rightarrow$ be the symbol for the counterfactual conditional. The condition on causation can then be expressed as follows: $(\neg C \Box \rightarrow \neg E) \wedge (C \Box \rightarrow E)$. It is this theory that was used by Koura (1988, p.196) in his theory about why-questions.

Before we continue, it is important to stop for a moment and reflect on the interpretation of the symbols. It is all good and well to have a theory that is about the causal relation between

events, but events are located in space and time and this restricts the applicability of the theory to the Question. Let us assume then that the counterfactual account has been properly generalized. I don't need to provide any details here: that is up for those who want to answer the Question.

The (simple) counterfactual approach to causation and, indirectly, explanation does not allow for non-question-begging answers to the Question. Suppose that there is a sentence ϕ such that $\phi \Box \rightarrow \exists x E!x$. Note that an important inference rule for counterfactual conditionals is the following (Lewis, 1973b, p. 27): if $\phi \Box \rightarrow \psi$, then $\phi \rightarrow \psi$. So, we also have that $\phi \rightarrow \exists x E!x$. As Koura (1988, p. 196) points out, a causal-counterfactual answer to a why-question requires that the antecedent of the counterfactual conditional is true. So, we also have ϕ . Then we can reason as before and deduce that $\phi \rightarrow \exists x \phi$ and, hence, $\exists x \phi$. One can then logically deduce that $\exists x (\phi \wedge \phi \Box \rightarrow \exists x E!x)$. Since the latter logically entails $\phi \wedge \phi \Box \rightarrow \exists x E!x$ (axiom scheme **A1***), the answer is again question-begging. This analysis presupposes negative or positive free logic. For an analysis that starts from neutral free logic one would need rules for $(\alpha \Box \rightarrow \beta)^{(*)}$ and $\neg(\alpha \Box \rightarrow \beta)^{(*)}$.

The second main alternative consists in probabilistic answers. The minimal version of this is that C explains E if and only if the probability of E conditional on C is higher than the unconditional probability of E . Let Pr be the symbol for probability functions. The condition can then be expressed as follows: $Pr(E | C) > Pr(E)$. It is this version that was used by Koura (1988, p. 197). Of course, the conditional probability of E on C might be low as long as the unconditional probability of E is lower still. Some philosophers think that this is too minimal. E.g., Salmon (1992, p. 33) remarks that statistical relevance, to which probability-raising belongs, can be used as evidence for causal relevance, but it is causal relevance that carries explanatory weight. Woodward (2014, Section 3.4) elaborates on this. Strevens (2000) claims that probabilistic explanations with higher probabilities are better, while he also suggests that in the case of low probabilities it is something else that is explanatorily significant. With these qualifications in mind, let us look at probabilistic answers to the Question.

An immediate problem is to find a suitable interpretation of the probabilities. The subjective interpretation of probability as degree of belief by a doxastic agent is not well-suited. For broadly speaking Cartesian considerations make it implausible that a doxastic agent does not assign probability one to the proposition that he exists.⁶ This would make probabilistic answers to the Question impossible. For if $Pr(\exists x E!x) = 1$, then there cannot be an answer ϕ such that $Pr(\exists x E!x | \phi) > Pr(\exists x E!x)$. Of course, there are alternative interpretations of probability, viz. quasi-logical and objective interpretations (Hájek, 2012). Let us assume for the sake of the argument that an interpretation of probability can be given that also makes sense of the Question. A further issue is then how to assign probabilities to the various possibilities, including the possibility that nothing exists. Kotzen (2013) discusses various difficulties with this. I will not go into these difficulties, but I want to point out that there is a common but debatable assumption that goes back at least to the contribution by Van Inwagen and Lowe (1996) to the debate. The assumption is that there is only one possible world with an empty universe. That assumption is all right if one presupposes negative free logic. But in positive free logic there are, for instance, at least two models with an empty domain of quantification, where the first model makes an atomic sentence true and the second model makes it false. The reason is that besides a possibly empty *inner* domain of quantification models for that logic have also a non-empty *outer* domain. The interpretation function can assign subsets of that outer domain to predicates.

Suppose that one can make sense of the probability in a probabilistic answer to the Question and that one has a grasp on how to assign the probabilities. Let ϕ be a sentence such that $Pr(\exists x E!x | \phi) > Pr(\exists x E!x)$. Normally it is postulated that, if $\vdash_{CL=} \phi$, then $Pr(\phi) = 1$. This postulate has to be replaced by the corresponding one for free logic. Note also that, if $\phi \vdash \psi$,

⁶I say 'broadly speaking', because Hintikka (1962) points to some problems that I will not elaborate on.

then $Pr(\phi) \leq Pr(\psi)$. Moreover, if $\phi \dashv\vdash \psi$, then $Pr(\phi) = Pr(\psi)$. Finally, we need some facts about conditional probability. First, if $\phi \vdash \neg\psi$, then $Pr(\psi | \phi) = 0$. Second, if $\phi \dashv\vdash \psi$, then $Pr(\phi | \theta) = Pr(\psi | \theta)$. Third, $Pr(\phi | \psi) = Pr(\phi \wedge \psi | \psi)$. Applying the law of total probability to $\exists x\phi$ yields the following:

$$Pr(\exists x\phi) = (Pr(\phi) \times Pr(\exists x\phi | \phi)) + (Pr(\neg\phi) \times Pr(\exists x\phi | \neg\phi)).$$

One can prove in free logic that $\exists x\phi$ is logically equivalent with $\exists xE!x \wedge \phi$. But then

$$Pr(\exists x\phi | \neg\phi) = 0.$$

So, $Pr(\exists x\phi) = Pr(\phi) \times Pr(\exists x\phi | \phi)$. Furthermore, note that the logical equivalence of $\exists x\phi$ and $\exists xE!x \wedge \phi$ entails that $Pr(\exists xE!x | \phi) = Pr(\exists x\phi | \phi)$. Next, assume that Hempel and Oppenheim (1948) are right about the fact that probabilistic explanations need to make the *explanandum* highly probable, or that Strevens (2004) is right that higher probabilities result in better explanations. We already knew that $Pr(\exists x\phi) \leq Pr(\phi)$. If $Pr(\exists xE!x | \phi)$ is higher and, therefore, on our assumption gives a better probabilistic explanation, then the difference between $Pr(\exists x\phi)$ and $Pr(\phi)$ is smaller. This argument may induce one to think that probabilistic answers to the Question may be in a sense question-begging as well, although I admit that the result is not as rock-solid as previously obtained results.

In this section the possibility of causal and probabilistic answers to the Question has been investigated. For the simplest type of causal-counterfactual answers and against a background of negative or positive free logic I have argued that any answers have to be question-begging. I have not surveyed all the types of causal-counterfactual answers nor have I looked at it from the perspective of neutral free logic. For probabilistic answers I have mentioned a couple of issues. They may ultimately have to be replaced by causal answers. It is not clear how to interpret the probabilities in this context. And one has to be careful in one's assignment of probabilities. Setting all that aside, I have argued that the higher the probability of there existing something conditional on the answer, the closer to equiprobability the answer and its existential counterpart are. The discussion in this section is much less comprehensive and the conclusions are more tentative than the discussion and conclusions in Sections 2 and 3.

6 Conclusion: Stop Asking The Question?

Leibniz's question 'Why is there something rather than nothing?' continues to attract attention. In this article I have undertaken a logical study of the Question. The starting point was the logic of why-questions. An answer to a why-question is an explanation. According to some prominent theories of explanation an explanation has a deductive component: at some point the presupposition of the question has to be deduced from something else.

The background logic cannot be classical first-order logic with identity, since it has as a theorem that at least one thing exists. Therefore any purported answer to the Question is trivial. In free logics it is not a theorem that at least one thing exists. Free logics come in three main varieties, viz. negative, positive and neutral. I have proved that, if negative free logic is the background logic, any argument with an existential sentence as conclusion has at least one premise that is provably equivalent to an existential sentence (Theorem 1). Next I have proved that, if positive free logic is in the background, any argument with an existential sentence as conclusion has at least one premise that is itself an existential sentence or there are premises such that their conjunction is provably equivalent to an existential sentences (Theorem 2). Then I have proved that, if neutral free logic (in its Fregean form) is in the background, any argument with an existential sentence as conclusion and truth-determinate sentences as premises has at

least one premise that is provably equivalent to an existential sentence (Theorem 3). All three main results are taken to imply that any answer to the Question is question-begging.

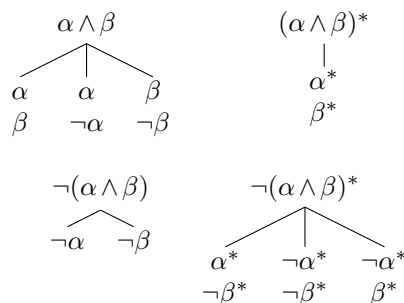
In addition I have looked at the Modal Question, viz. ‘Why is there something contingent rather than nothing contingent?’. If the deductive framework is classical, the presupposition of the question is logically false, which precludes sound arguments for it. If the deductive framework is free logic in its positive or negative variety, then any purported answer is question-begging (Theorem 4). The possibility of answering the modal version of the Question against the backdrop of neutral free logic has not been investigated. Furthermore I have discussed the Categorical Question, viz. ‘Why is there something concrete rather than nothing concrete?’. Here we have found that deductive explanations are question-begging, whether one assumes positive or negative free logic or classical logic (Theorem 5).

In Section 5 the assumption that answers to why-questions have a deductive component was dropped. Two major alternatives were considered, namely ‘causal’ or rather counterfactual answers and probabilistic answers. In both cases there are interpretational difficulties, but setting those aside I provided two reasons to be skeptical. On a simple counterfactual analysis answers to the Question have to be question-begging. Probabilistic answers are better to the extent that the probability of the existentially quantified version of the answer is closer to the answer itself, which has a whiff of circularity around it. The discussion and conclusions of the Section 5 were much less comprehensive and much more tentative than before but the outlook remained negative.

On the assumption that answers to why-questions need to have a deductive component, the conclusion is that neither the Question nor the Modal Question nor the Categorical Question can be answered adequately. (But recall that the possibility of answers to the modal and the categorial versions of the Question against the background of neutral free logic have not been studied.) The meta-question is then naturally: should we not stop asking the Question and its ilk? According to Searle (1969) the point of asking questions is to request something. In the case of why-questions (a type of) explanations is requested for. If one knows that a request cannot be met, it is pointless to keep requesting it. So, the rational answer to the meta-question is positive. If the underlying assumption is dropped and the possibility of ‘causal’ or probabilistic answers to the Question are considered, then it *may* still be rational to ask the Question, although there are reasons to be skeptical in this case as well.

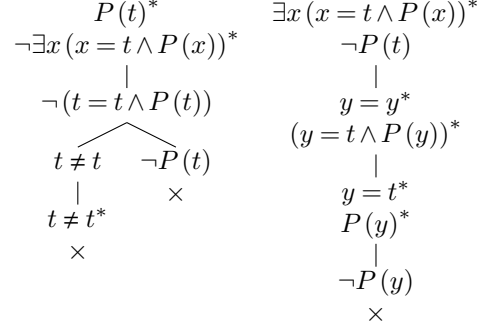
A Tree proofs for Lemma 2

Before giving the tree proofs, let me remind the reader that $(\alpha \wedge \beta)$ is definitionally equivalent to $\neg(\alpha \rightarrow \neg\beta)$ and, consequently, $\neg(\alpha \wedge \beta)$ is definitionally equivalent to $\neg\neg(\alpha \rightarrow \neg\beta)$, which logically entails $(\alpha \rightarrow \neg\beta)$ (cf. the tree rules for double negation). So, one can use the rules for material implications.



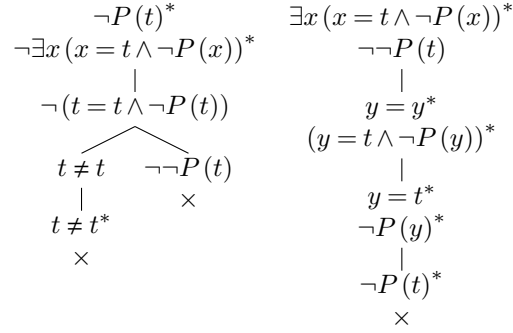
For convenience, whenever the *reductio* assumption of a tree proof is a quantified sentence or the negation thereof the marker $*$ will be added directly.

Case 1-i. For convenience and without loss of generality, let us consider only $P(t)^*$ and $\exists x(x = t \wedge P(x))^*$.

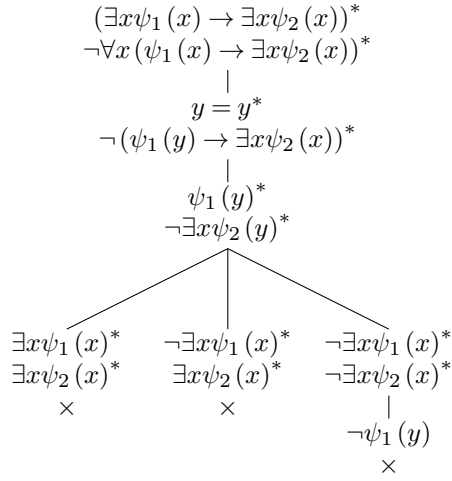


□

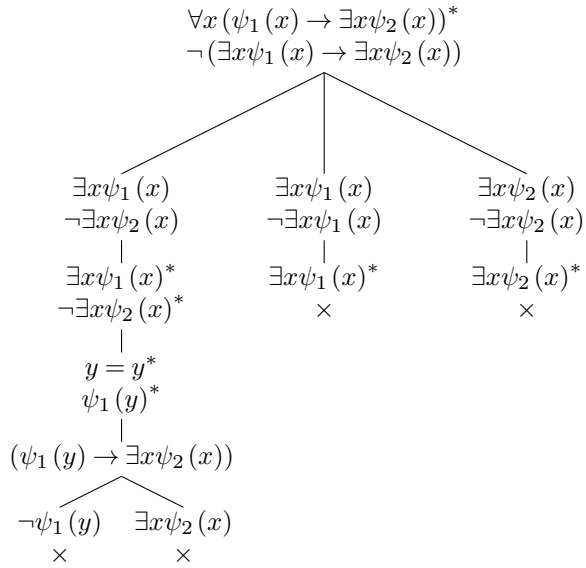
Case 1-ii. For convenience and without loss of generality, let us consider only $P(t)^*$ and $\exists x(x = t \wedge P(x))^*$.



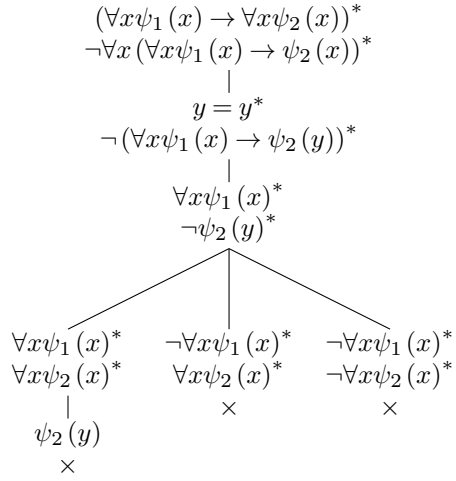
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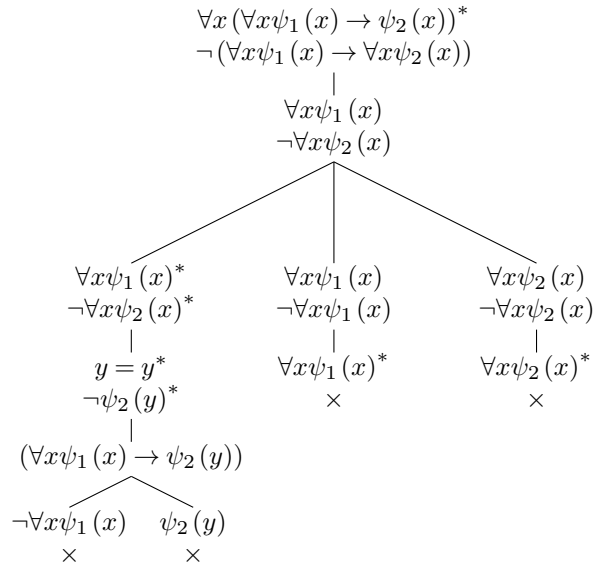
Case 3.1.



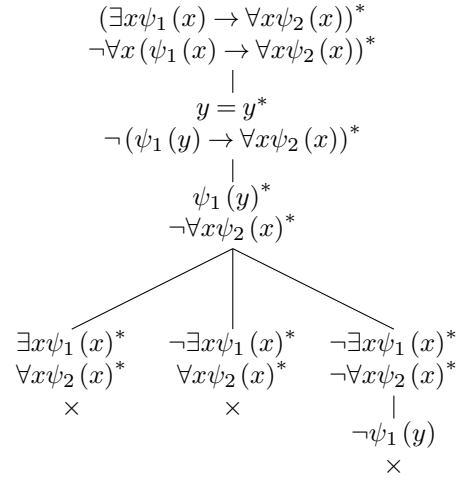
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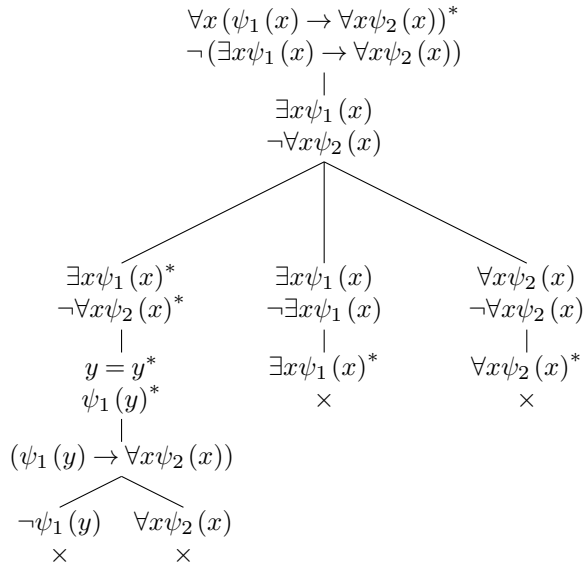
Case 3.2.



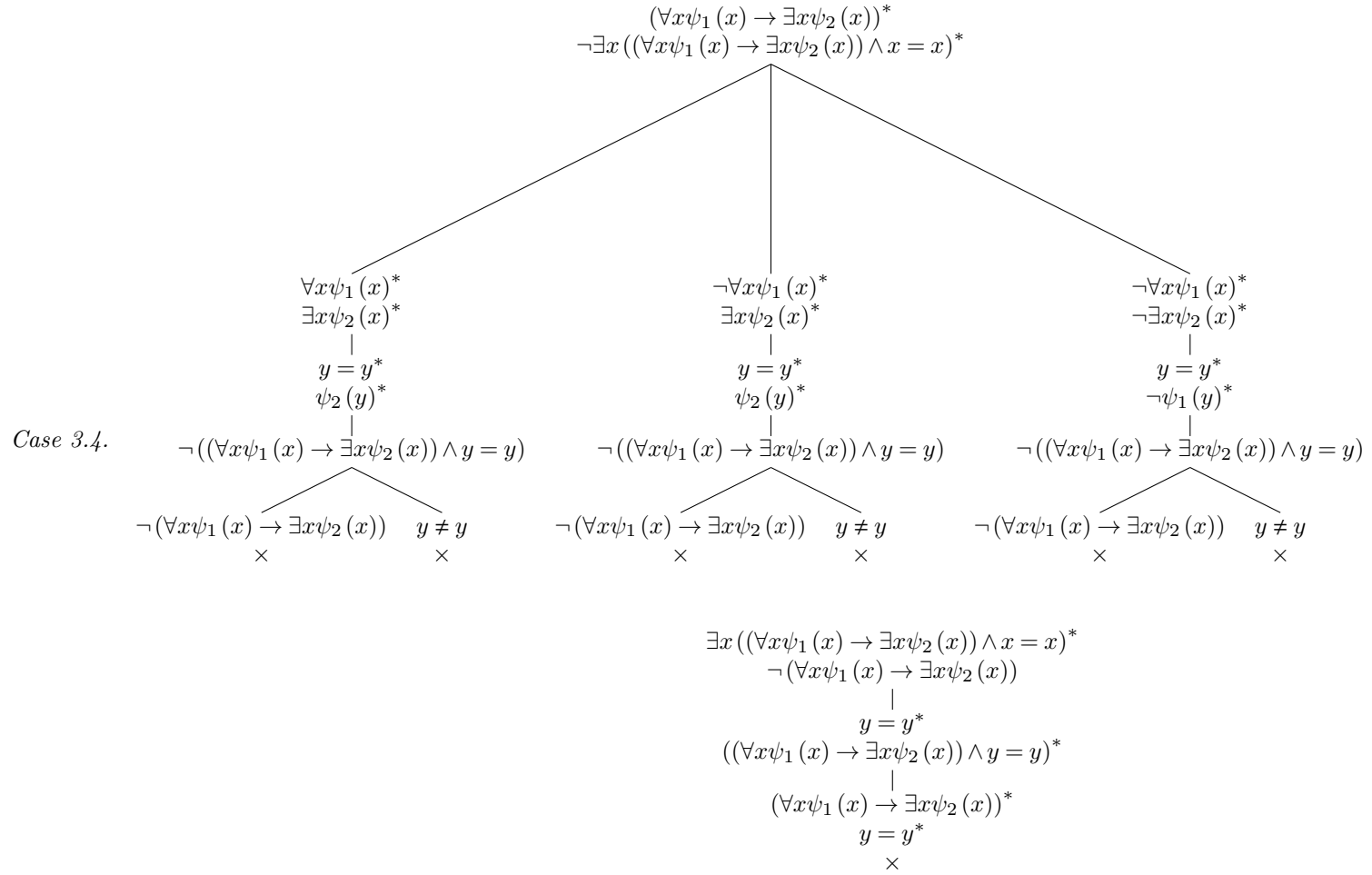
□



Case 3.3.



□



□

$$\begin{array}{c}
\neg(\exists x\psi_1(x) \rightarrow \exists x\psi_2(x))^* \\
\neg\exists x\neg(\psi_1(x) \rightarrow \exists x\psi_2(x))^* \\
| \\
\exists x\psi_1(x)^* \\
\neg\exists x\psi_2(x)^* \\
| \\
y = y^* \\
\psi_1(y)^* \\
| \\
\neg\neg(\psi_1(y) \rightarrow \exists x\psi_2(x)) \\
| \\
\psi_1(y) \rightarrow \exists x\psi_2(x) \\
\swarrow \quad \searrow \\
\neg\psi_1(y) \quad \exists x\psi_2(y) \\
\times \qquad \times
\end{array}$$

Case 3.1-ii.

$$\begin{array}{c}
\exists x\neg(\psi_1(x) \rightarrow \exists x\psi_2(x))^* \\
\neg\neg(\exists x\psi_1(x) \rightarrow \exists x\psi_2(x)) \\
| \\
y = y^* \\
\neg(\psi_1(y) \rightarrow \exists x\psi_2(x))^* \\
| \\
\psi_1(y)^* \\
\neg\exists x\psi_2(x)^* \\
| \\
\exists x\psi_1(x) \rightarrow \exists x\psi_2(x) \\
\swarrow \quad \searrow \\
\neg\exists x\psi_1(x) \quad \exists x\psi_2(x) \\
| \qquad \qquad \times \\
\neg\exists x\psi_1(x)^* \\
| \\
\neg\psi_1(y) \\
\times
\end{array}$$

□

$$\begin{array}{c}
\neg(\forall x\psi_1(x) \rightarrow \forall x\psi_2(x))^* \\
\neg\exists x\neg(\forall x\psi_1(x) \rightarrow \psi_2(x))^* \\
| \\
\forall x\psi_1(x)^* \\
\neg\forall x\psi_2(x)^* \\
| \\
y = y^* \\
\neg\psi_2(y)^* \\
| \\
\neg\neg(\forall x\psi_1(x) \rightarrow \psi_2(y)) \\
| \\
\forall x\psi_1(x) \rightarrow \psi_2(y) \\
\swarrow \quad \searrow \\
\neg\forall x\psi_1(x) \quad \psi_2(y) \\
\times \qquad \qquad \times
\end{array}$$

Case 3.2-ii.

$$\begin{array}{c}
\exists x\neg(\forall x\psi_1(x) \rightarrow \psi_2(x))^* \\
\neg\neg(\forall x\psi_1(x) \rightarrow \forall x\psi_2(x)) \\
| \\
y = y^* \\
\neg(\forall x\psi_1(x) \rightarrow \psi_2(x))^* \\
| \\
\forall x\psi_1(x)^* \\
\neg\psi_2(y)^* \\
| \\
\forall x\psi_1(x) \rightarrow \forall x\psi_2(x) \\
\swarrow \quad \searrow \\
\neg\forall x\psi_1(x) \quad \forall x\psi_2(x) \\
\times \qquad \qquad | \\
\qquad \qquad \psi_2(y) \\
\qquad \qquad \times
\end{array}$$

□

$$\begin{array}{c}
\neg(\exists x\psi_1(x) \rightarrow \forall x\psi_2(x))^* \\
\neg\exists x\neg(\exists\psi_1(x) \rightarrow \psi_2(x))^* \\
| \\
\exists x\psi_1(x)^* \\
\neg\forall x\psi_2(x)^* \\
| \\
y = y^* \\
\neg\psi_2(y)^* \\
| \\
\neg\neg(\exists\psi_1(x) \rightarrow \psi_2(y)) \\
\swarrow \quad \searrow \\
\neg\exists x\psi_1(x) \quad \psi_2(y) \\
\times \quad \times
\end{array}$$

Case 3.3-ii.

$$\begin{array}{c}
\exists x\neg(\exists x\psi_1(x) \rightarrow \psi_2(x))^* \\
\neg\neg(\exists x\psi_1(x) \rightarrow \forall x\psi_2(x)) \\
| \\
y = y^* \\
\neg(\exists x\psi_1(x) \rightarrow \psi_2(y))^* \\
| \\
\exists x\psi_1(x)^* \\
\neg\psi_2(y)^* \\
| \\
\exists x\psi_1(x) \rightarrow \forall x\psi_2(x) \\
\swarrow \quad \searrow \\
\neg\exists x\psi_1(x) \quad \forall x\psi_2(x) \\
\times \quad | \\
\quad \psi_2(y) \\
\quad \times
\end{array}$$

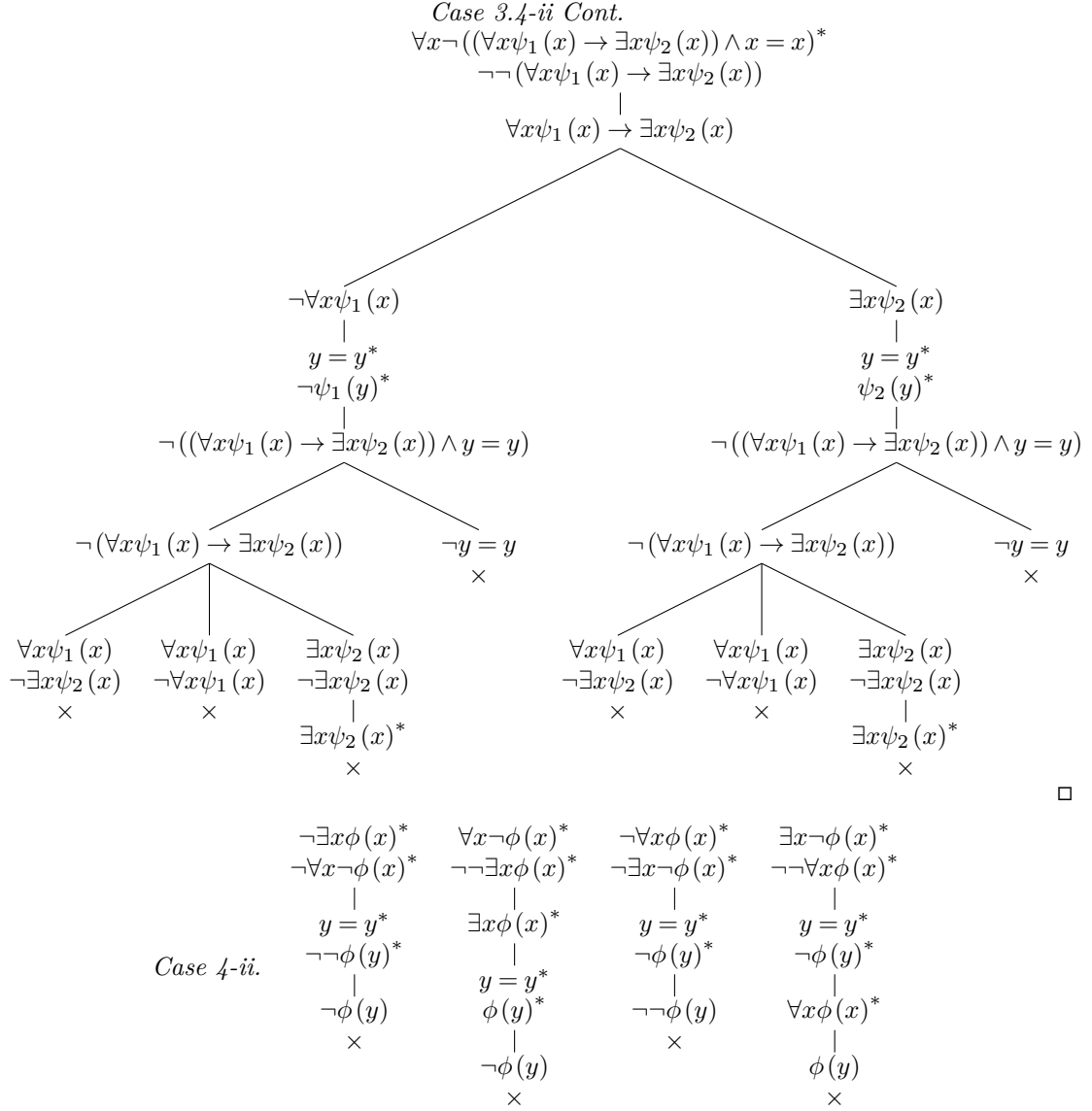
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$$\begin{array}{c}
\neg(\forall x\psi_1(x) \rightarrow \exists x\psi_2(x))^* \\
\neg\forall x\neg((\forall x\psi_1(x) \rightarrow \exists x\psi_2(x)) \wedge x = x)^* \\
| \\
y = y^* \\
\neg\neg((\forall x\psi_1(x) \rightarrow \exists x\psi_2(x)) \wedge y = y)^* \\
| \\
(\forall x\psi_1(x) \rightarrow \exists x\psi_2(x))^* \\
y = y^*
\end{array}$$

Case 3.4-ii.

$$\begin{array}{c}
\forall x\psi_1(x)^* \\
\neg\exists x\psi_2(x)^* \\
\swarrow \quad \downarrow \quad \searrow \\
\forall x\psi_1(x)^* \quad \neg\forall x\psi_1(x)^* \quad \neg\forall x\psi_1(x)^* \\
\exists x\psi_2(x)^* \quad \exists x\psi_2(x)^* \quad \neg\exists x\psi_2(x)^* \\
\times \quad \times \quad \times
\end{array}$$

□



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